The effect of gas density on fluidized-bed entrainment

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Keywords: fluidization, entrainment, gas density, pressure, helium, air, cold models, 2D column, 3D column

Abstract—This article reports on an experimental study where the effect of gas density on fluidized bed entrainment was investigated using cold models. Fluidization experiments were conducted in a 3D cylindrical column in air at gas densities between 1.8–8.0 kg/m³ and superficial gas velocities up to 0.7 m/s. The experimental data was correlated with \[ E = 2.91 \rho_g U_g^{4.61} \] where \( E \) is the entrainment flux in kg/(m²·s), \( \rho_g \) is the gas density in kg/m³ and \( U_g \) is the superficial gas velocity in m/s. The entrainment model of Zenz and Weil⁶ was also found to provide a good fit of the experimental entrainment rate data obtained with air in this study.

A second set of experiments was performed in a 2D rectangular column using both helium (0.17 kg/m³) and air (1.20 kg/m³) at ambient conditions up to velocities of 1 m/s. Surprisingly, similar entrainment rates were observed when using helium and air as fluidization gas. The similar observed entrainment rates in air and helium were confirmed in a couple of experiments in the 3D column. A simple single particle model showed that in the laminar regime, the terminal velocity is not affected by the gas density below a certain value. Based on the experimental data, this threshold value of the gas density is arguably close to the gas density of air at atmospheric pressure (1.20 kg/m³) for the tested fluidized bed material.

INTRODUCTION

Sasol is an international integrated energy and chemical company originally based in Sasolburg, South-Africa. Sasol develops and commercializes technologies, and builds and operates world-scale facilities to produce a range of product streams, including liquid fuels, high-value chemicals and electricity.¹

Gas-solid fluidization finds wide application within Sasol e.g. at their Secunda plants, high pressure fluidized bed reactors called Sasol Advanced Synthol (SAS) reactors are used to convert syngas to liquid fuels and chemicals.² This article reports on a cold model study carried out at Sasol to investigate the effect of gas density (0.17–8.0 kg/m³) on the entrainment rate in a gas-solid fluidized bed.

THEORY

The effect of gas density on single particle terminal velocity

The terminal fall velocity for a single particle and entrainment rate are likely to be correlated to each other but it is important to note that the effect of the surrounding particles or particle agglomerates is completely neglected within this single particle analysis. A correlation for the
terminal fall velocity for a single particle (acceleration is zero) was derived from a force balance around a single particle:

\[ F_L + F_D = F_g \]  \hspace{1cm} (1)

\[ \frac{1}{6} \pi d_p^3 \rho_g g + C_D \frac{\pi}{4} d_p^2 \rho_g \frac{U_d^2}{2} = \frac{1}{6} \pi d_p^3 \rho_p g \]  \hspace{1cm} (2)

Assuming \( \rho_g << \rho_p \)

\[ U_u = \frac{8 \ d_p \rho_p}{\sqrt{6 \ C_D \rho_g}} \]  \hspace{1cm} (3)

The drag coefficient, \( C_D \), is dependent on the Reynolds number and consequently on the gas density.

In the Stokes regime: \( \text{Re} < 0.25 \)

\[ C_D = \frac{24}{\text{Re}} \text{Re} = 24 \frac{\eta_g}{\rho_g U_g d_p} \sim \frac{1}{\rho_g} \to U_u \text{ not dependent on } \rho_g \]  \hspace{1cm} (4)

In the transition regime: \( 0.1 < \text{Re} < 4 \times 10^3 \) (from Kürten):

\[ C_D = \frac{21}{\text{Re}} + \frac{6}{\sqrt{\text{Re}}} + 0.28 = 21 \frac{\eta_g}{\rho_g U_g d_p} + 6 \sqrt{\frac{\eta_g}{\rho_g U_g d_p}} + 0.28 \to U_u \text{ dependent on } \rho_g \]  \hspace{1cm} (5)

The gas density does not have an effect on the terminal fall velocity (entrainment) in the Stokes regime (\( \text{Re} < 0.25 \)). In conclusion the single particle model shows that within the laminar regime, a gas density cut-off point is likely to exist below which the entrainment rate is not affected by the gas density anymore. However this cut-off point has not been reported in literature for the entrainment in fluidized beds to the best of the author’s knowledge (see next section).

The effect of gas density on fluid bed entrainment

There have been a large number of literature studies on entrainment in fluidized beds. Overviews including the published entrainment rate correlations are given in the book of Kunii and Levenspiel and in the handbook of Wen-Ching Yang. Two of the correlations are given in Table 1: i) Zenz and Weil (1958), and ii) Tasirin and Geldart (1998). These correlations were selected for comparison because similar experimental conditions were applied in this study.

In literature it is reported that the entrainment rate is increasing with gas density. Yates (1996) reported that the entrainment rate was linearly proportional to pressure (and consequently gas density) up to 21 bar. The same dependency on gas density holds for the correlations developed by Zenz & Weil (1958) and Tasirin & Geldart (1998) (see Table 1).

EXPERIMENTAL PROCEDURE

Cold models

3D-column: \( 0.17 < \rho_g < 8.0 \text{ kg/m}^3 \)

A schematic overview of the 3D bed is shown in Figure 1 (right). The column is made of stainless steel and has the following dimensions: a diameter of 15 cm and a total height of 4.25 m. Air and helium were used as fluidization gas. For the experiments performed in air the pressure was varied between 1.5 bara and 6.7 bara. Superficial gas velocities between 0.03 to 0.7 m/s were used. The gas was fed via a perforated plate. The plate consisted of three layers i)
perforated plate \((d_h=2 \text{ mm}, \text{open area}=2.5\%)\) ii) wire-mesh \((40 \mu\text{m})\) iii) again the same perforated plate. A variable area flowmeter from Brooks Instruments was used for the measurement of the helium flow rate and a flow controller (Fisher SN EU03651815) connected in series with a mass flow meter (Endress and Hauser, Promass F) was used to measure the air flow rate. The column was filled with solids up to a static bed height of 100 cm. The solids leaving the column were collected in a cyclone and filter. After completing the experiment the solids were returned to the column before starting the next run.

### Table 1. Literature correlations for elutriation constant \(K_{ic}^*\) [kg/(m²·s)]

<table>
<thead>
<tr>
<th>Ref</th>
<th>Correlation</th>
<th>Conditions</th>
</tr>
</thead>
</table>
| Zenz and Weil 1958 | \[ K_{ic}^* = \left\{ \begin{array}{l} 1.26 \times 10^7 \rho_g U_s \left( \frac{U_s}{gd_p \rho_p} \right)^{1.38} \\
                           \text{for} \quad \frac{U_s^2}{gd_p \rho_p^2} < 3 \times 10^{-4} \\
                           4.31 \times 10^4 \rho_g U_s \left( \frac{U_s}{gd_p \rho_p} \right)^{1.18} \\
                           \text{for} \quad \frac{U_s^2}{gd_p \rho_p^2} > 3 \times 10^{-4} \end{array} \right. \] | \[
\begin{align*}
0.3 \text{ m/s} & < 0.7 \text{ m/s} \\
5 \text{ cm} & < D < 53 \text{ cm} \\
\text{FCC-catalyst} & \\
40 < d_p & < 200 \mu\text{m} \\
\rho_k & = 1.2-17.6 \text{ kg/m}^3
\end{align*}\] |
| Tasirin and Geldart 1998 | \[ K_{ic}^* = \left\{ \begin{array}{l} 23.7 \cdot \rho_g \cdot U_s^{2.3} \cdot e^{-5 \frac{d_p}{U_s}} \\
                           \text{for} \quad \text{Re} < 3000 \\
                           14.5 \cdot \rho_g \cdot U_s^{2.5} \cdot e^{-5 \frac{d_p}{U_s}} \\
                           \text{for} \quad \text{Re} > 3000 \end{array} \right. \] | \[
\begin{align*}
0.3 \text{ m/s} & < 0.7 \text{ m/s} \\
7.6 \text{ cm} & < D < 15.2 \text{ cm} \\
\text{FCC-catalyst} & \\
17 < d_p & < 77 \mu\text{m} \\
\rho_k & = 1.2 \text{ kg/m}^3
\end{align*}\] |

### 2D-column: \(0.17 < \rho_g < 1.2 \text{ kg/m}^3\)
A schematic overview of the 2D bed is shown in Figure 1 (left). The 2D bed is made of transparent Lexaan and has the following dimensions: thickness 2 cm, width 46 cm and a total height of 3.0 m. The gas was fed via a filter cloth \((25 \mu\text{m})\) and a perforated plate \((d_h=1 \text{ mm}, \text{open area}=1\%)\) via three differently sized variable flow meters (from Brooks instruments). Experiments were carried out at superficial gas velocities between 0.1 and 1 m/s. The column was filled with solids up to a static bed height of 30–110 cm. The solids leaving the column were collected in a cyclone and filter placed in series \((\text{pore size }1\mu\text{m})\).

A disadvantage of using a 2D bed is that the effect of the wall cannot be excluded. Despite this it was chosen to perform experiments in this set-up because of i) visual access and ii) less gas (helium) is required.

### Materials
A powder having a particle density of about 2000 kg/m³ and a Sauter mean diameter of around 90 \(\mu\text{m}\) was used as fluidized bed material. Helium \((\sim P_{atm})\) and air \((P_{atm} - 6.7 \text{ bara})\) were used as fluidization gas. The gas density of helium is significantly lower than that of air, but the viscosities are similar. The gas densities and viscosities are listed in Table 2.

### Table 2. Properties air and helium at 20 °C

<table>
<thead>
<tr>
<th>Gas</th>
<th>(\rho_g) [kg/m³]</th>
<th>(\eta) [Pa·s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium (Patm)</td>
<td>0.2</td>
<td>0.0000196</td>
</tr>
<tr>
<td>Air (Patm)</td>
<td>1.2</td>
<td>0.0000182</td>
</tr>
<tr>
<td>Air (6.7 bara)</td>
<td>8.0</td>
<td>0.0000182</td>
</tr>
</tbody>
</table>
Analytical techniques

The total amount of entrained material is defined as the sum of the solids collected in the cyclone and filter (Figure 1). The amount of solids collected in the filter for the 2D column (not 3D column) was neglected. Less than 0.1%wt. of the entrained solids was collected in this filter. The entrainment rate \([\text{kg}/(\text{m}^2 \cdot \text{s})]\) was subsequently calculated using eq. 6.

\[
E = \frac{\text{mass solids in cyclone} + \text{mass solids in filter}}{\text{run time} \cdot A_{\text{reactor}}} \tag{6}
\]

EXPERIMENTAL RESULTS

Air at different pressures: \(1.8 < \rho_g < 8.0 \text{ kg/m}^3\)

A few experiments were carried out initially to confirm that the measured entrainment rate was above the transport disengagement height (TDH). This was carried out by measuring the entrainment rate obtained at varying static bed heights (e.g. 50 cm versus 100 cm). This initial exercise showed that the measured entrainment rate was indeed above the TDH (same entrainment rate obtained for the two static bed heights used at a fixed superficial gas velocity) and that the repeatability was also acceptable. The measured entrainment rate obtained for the 3D column as a function of velocity and gas density/pressure is plotted in Figure 2A and 2B.
A) Entrainment rate as a function of superficial gas velocity, Trend lines: $E = 2.91 \rho_g U_g^{4.61}$

B) Entrainment rate as a function of gas density/pressure, Trend lines: Excel linear

Figure 2. Measured entrainment results as a function of (A) superficial gas velocity and (B) gas density/pressure (experiments were carried out in a 3D column using air as fluidization gas and pressures between 1.5 and 6.7 bara)

The entrainment rate increased with a power law relationship with superficial gas velocity. The power of the correlating equation appeared to be independent of gas density/pressure and its value was 4.61 (Figure 2A and Equation 7). It is important to note that the entrainment rate was increasing with pressure at the same superficial gas velocity. A linear increase with pressure at the same superficial gas velocity was observed. The effect of pressure on the entrainment rate was more pronounced at higher superficial gas velocities (Figure 2B). It was possible to fit the measured entrainment rate ($E$ in kg/(m²·s)) data using a power function with respect to velocity ($U_g$ in m/s) and a linear function with respect to gas density ($\rho_g$ in kg/m³). The result is shown in equation 7 for gas densities in the range 1.8 and 8.0 kg/m³.

$$E = 2.91 \cdot \rho_g \cdot U_g^{4.61} \quad (7)$$

Helium versus air: $\rho_g \sim 0.17$ and $1.2$ kg/m³

Based on the data obtained with air in the 3D column above, lower entrainment rates were expected when fluidizing the bed with helium. Based on the density difference between helium and air at atmospheric conditions it was expected that the entrainment rate in helium would be approximately lower by a factor 7 when compared to entrainment with air fluidization ($\rho_{air}/\rho_{helium} \sim 7$).

Entrainment rate measurements were therefore carried out in the 2D column (to minimize helium consumption) to confirm the above. These results are presented in Figure 3. Surprisingly it was found that similar entrainment rates were observed in the 2D column when using helium and air as fluidization gas.

For both gases the entrainment rate was found to increase with a power law relationship on the superficial gas velocity raised to the power 6. Similar entrainment rates were observed using
helium and air as fluidization gas for all tested disengagement heights. The disengagement height was about 1.8 m at 0.65 m/s and 2.3 m at 0.81 m/s as shown in Figure 3B.

To further confirm the above finding, a couple of experiments were repeated in the 3D column where the entrainment rate was measured with air and helium at a pressure of 1.5 bara. The measured entrainment rates as a function of superficial gas velocity in the 3D column using air ($\rho_g=1.8$ kg/m$^3$) and helium ($\rho_g=0.26$ kg/m$^3$) as fluidization gas are plotted in Figure 4. These experiments confirmed the observations reported earlier in the 2D bed as no difference in entrainment between helium and air could be observed in the 3D bed.

**DISCUSSION**

**Tasirin & Geldart and Zenz & Weil models**

The experimental data obtained in the 3D column was compared with the model predictions of Tasirin & Geldart and Zenz & Weil. The results are summarized in Figure 5.

In line with the model of Zenz & Weil and Tasirin & Geldart it was experimentally observed that the entrainment rate was linearly proportional to gas density in the pressure range tested (1.5–6.7 bara in air). The observed power-law dependency on gas velocity is also supported with the model of Zenz & Weil. The absolute experimental entrainment rate data appeared to be extremely close to the model of Zenz & Weil but generally over-predicted by the model of Tasirin & Geldart (in the pressure range 1.5–6.7 bara in air).
Figure 4. Measured entrainment results as a function of superficial gas velocity (experiments carried out in the 3D column at 1.5 bara using air and helium as fluidization gas. Trend line: \( E = 2.91 \cdot 1.8 \cdot U_g^{4.61} \) – see Eq. 7)

Figure 5. Comparison of measured entrainment rates in the 3D column with model predictions of Tasirin & Geldart and Zenz & Weil (see Table 1), Trend lines experimental entrainment rate: \( E = 2.91 \rho \rho g U_g^{4.61} \) (Eq. 7)

The model of Tasirin & Geldart, the model of Zenz & Weil and the extrapolation of equation 10 proposed in this article were in conflict with the similar observed entrainment rates at lower gas densities (helium and air experiments). According to these models the entrainment rate in helium is expected to be a factor 7 lower than in air (\( \rho_{\text{air}} / \rho_{\text{helium}} \sim 7 \)). However none of the correlations (except for equation 7) were validated for lower gas densities than air at
atmospheric pressure (1.2 kg/m³). This finding highlights further the danger in extrapolating literature entrainment correlations to process conditions that are outside the region in which the correlations were developed.

**Single particle force balance**

On the basis of the single particle model it was shown earlier in the “theory” section that a gas density cut-off point is likely to exist below which the terminal fall velocity (entrainment rate) should not be affected by the gas density anymore in the laminar regime.

Although more work is required to understand this phenomenon in more detail, it appears that for the powder tested in this work the cut-off density below which the entrainment rate is independent of gas density approximately equals to that of air at atmospheric pressure (1.2 kg/m³).

**CONCLUSIONS**

The entrainment rate of a powder was investigated using 2D and 3D fluidization columns which utilized both air and helium as fluidization gas. It was found that with air fluidization the measured entrainment rate was found to increase linearly with gas density as the gas density was increased from 1.2 kg/m³ to 8.0 kg/m³. The entrainment rate increase followed a power law relationship with superficial gas velocity. The power was found to be independent of the gas density. As a result it was possible to fit the measured entrainment rate (E in kg/(m²·s)) data using a power function with respect to velocity (U in m/s) and a linear function with respect to gas density (ρg in kg/m³), i.e. E = 2.91ρgUg4.61. This correlation appeared not to hold anymore for gas densities lower than air at atmospheric pressure (1.20 kg/m³). Similar entrainment rates were observed when using helium (ρg = 0.17 kg/m³) and air as fluidization gas.

The entrainment model of Zenz & Weil fitted the experimental data of this work reasonably well for the gas densities between 1.8 and 8.0 kg/m³. When extrapolating the models to gas densities below that of air at ambient conditions it was found that the entrainment models did not predict the measured entrainment rates correctly.

A simple single particle model was used to show that in the laminar regime, the terminal fall velocity should not be affected by the gas density below a certain value. Based on the experimental data, this threshold value of the gas density is arguably close to the gas density of air at atmospheric pressure (1.20 kg/m³) for the tested fluidized bed material It was also encouraging to illustrate through this work the application of simple models to show directional changes and to further emphasize the dangers in extrapolating literature entrainment correlations to process conditions outside the bounds on which they were developed.

**ACKNOWLEDGMENTS**

The authors would like to acknowledge Berthold Breman, Joris Smit and Wimpie Booysen for the theoretical discussions, Ruben Oortman for modifying the experimental set-ups and bachelor student’s Tom van Ast and Remco Ongena for executing part of the experimental program.

**REFERENCES**


**NOMENCLATURE**

$A_{\text{reactor}}$: column cross-sectional area, $m^2$

$C_D$: drag coefficient

$D_{\text{reactor}}$: column internal diameter, $m$

$d_{p(i)}$: particle diameter (of particle $i$), $m$

$E$: entrainment rate, $kg/(m^2 \cdot s)$

$F_L$: lifting force (also called buoyancy force), $N$

$F_D$: drag force, $N$

$F_G$: gravity force, $N$

$g$: gravitational constant (9.81), $m/s^2$

$H_{\text{stat}}$: static bed height, $m$

$K$: elutriation constant, $kg/(m^2 \cdot s)$

$P$: pressure, $Pa$

$P_{\text{atm}}$: atmospheric pressure, $Pa$

$Re$: Reynolds number

$U_g$: superficial gas velocity, $m/s$

$U_t$: terminal fall velocity (also called elutriation velocity), $m/s$

$x_i$: volumetric fraction of particle $i$

Greek letters

$\eta_g$: gas viscosity, $kg/(m \cdot s)$

$\rho_g$: gas density, $kg/m^3$

$\rho_p$: particle density, $kg/m^3$